Two-Loop $\mathcal{O}(\alpha_s G_F m_t^2)$ Corrections to the Fermionic Decay Rates of the Standard-Model Higgs Boson

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Abstract

Low- and intermediate mass Higgs bosons decay preferably into fermion pairs. The one-loop electroweak corrections to the respective decay rates are dominated by a flavour-independent term of $\mathcal{O}(G_F m_t^2)$. We calculate the two-loop gluon correction to this term. It turns out that this correction screens the leading high-m_t behaviour of the one-loop result by roughly 10%. We also present the two-loop QCD correction to the contribution induced by a pair of fourth-generation quarks with arbitrary masses. As expected, the inclusion of the QCD correction considerably reduces the renormalization-scheme dependence of the prediction.

1 Introduction

One of the great puzzles of elementary particle physics today is whether nature makes use of the Higgs mechanism of spontaneous symmetry breaking to generate the observed particle masses. The Higgs boson, H, is the missing link sought to verify this concept in the Standard Model. Many of the properties of the Higgs boson are fixed, e.g., its couplings to the gauge bosons, $g_{VVH} = 2^{5/4} G_F^{1/2} M_V^2$ ($V = W, Z$), and fermions, $g_{f\bar{f}H} = 2^{1/4} G_F^{1/2} m_f$, and the vacuum expectation value, $v = 2^{-1/4} G_F^{-1/2} \approx 246$ GeV. However, its mass, M_H, and its self-couplings, which depend on M_H, are essentially unspecified.

The failure of experiments at LEP 1 and SLC to detect the decay $Z \to f\bar{f}H$ has ruled out the mass range $M_H \leq 63.8$ GeV at the 95% confidence level [1]. At the other extreme, unitarity arguments in intermediate-boson scattering at high energies [2] and considerations concerning the range of validity of perturbation theory [3] establish an upper bound on M_H at $\left(8\pi\sqrt{2}/3G_F\right)^{1/2} \approx 1$ TeV in a weakly interacting Standard Model.

The Higgs-boson discovery potential of LEP 1 and SLC is almost exhausted [4]. Prior to the advent of the LHC, the Higgs-boson search will be restricted to the lower mass range. With LEP 2 it should be possible to find a Higgs boson with $M_H \leq 100$ GeV when high energy and luminosity can be achieved [5]. A possible 4-TeV upgrade of the Tevatron might cover the M_H range up to 120 GeV or so [6]. At an e^+e^- linear collider operating at 300 GeV, 50 fb^{-1} luminosity and a b-tagging efficiency of 50% would be sufficient to detect a Higgs boson with $M_H \leq 150$ GeV in the $\mu^+\mu^- b\bar{b}$ channel [7].

Below the onset of the W^+W^- threshold, the Standard-Model Higgs boson is relatively long-lived, with $\Gamma_H < 100$ MeV, so that, to good approximation, its production and decay processes may be treated independently. The low-mass Higgs boson, with $M_H \leq M_Z$, decays with more than 99% probability into a fermion pair [8]. With M_H increasing, the W^+W^- mode, with at least one W boson being off shell, gradually gains importance. Its branching fraction surpasses that of the $\tau^+\tau^-$ mode at $M_H \approx 115$ GeV and that of the $b\bar{b}$ mode at $M_H \approx 135$ GeV [8]. In the near future, however, Higgs-boson searches will rely mostly on the $f\bar{f}$ modes.

Quantum corrections to Higgs-boson phenomenology have received much attention in the literature; for a review, see Ref. [9]. The experimental relevance of radiative corrections to the $f\bar{f}$ branching fractions of the Higgs boson has been emphasized recently in the context of a study dedicated to LEP 2 [5]. Techniques for the measurement of these branching fractions at a $\sqrt{s} = 500$ GeV e^+e^- linear collider have been elaborated in Ref. [10]. The QCD corrections to the $H \to q\bar{q}$ decay rates are most significant numerically [11]. In the approximation $m_q \ll M_H$, they are known to $\mathcal{O}(\alpha_s^2)$ [12]. The theoretical uncertainty related to the lack of knowledge of the terms of $\mathcal{O}\left(\alpha_s^2 m_q^2/M_H^2\right)$ and $\mathcal{O}(\alpha_s^3)$ is presumably small [13]. The bulk of the QCD corrections is attributed to the running of m_q up to scale M_H. In the case of the $b\bar{b}$ mode, the QCD correction relative to the Born approximation implemented with the pole mass ranges between -53% and -63% for M_H between 60 GeV and $2M_W$ [9].

The leading high-M_H correction to the $H \to f\bar{f}$ decay widths is flavour independent. The one-loop term, of $\mathcal{O}(G_F M_H^2)$, was first obtained by Veltman [14]; it is positive and reaches 11% at $M_H = 1$ TeV. Recently, the two-loop $\mathcal{O}(G_F M_H^2)$ term has been found [15]; it is negative and exceeds in magnitude the $\mathcal{O}(G_F M_H^2)$ term already at $M_H \approx 400$ GeV. The leading contributions due to new heavy fermions are also independent of the final-state flavour; at one loop, they are positive and increase quadratically with the heavy-fermion masses [16]. The full one-loop electroweak corrections to the $H \to f\bar{f}$ decay widths are now well established [17,18]. They consist of an electromagnetic and a weak part, which are separately finite and gauge independent. The electromagnetic part emerges from the one-loop QCD correction [11] by substituting αQ_f^2 for $\alpha_s C_F$, where Q_f is the electric charge of f and $C_F = (N_c^2 - 1)/(4N_c)$, with $N_c = 3$. For $M_H \ll 2M_W$, the weak part is well approximated by [18]

$$\Delta_{\text{weak}} = \frac{G_F}{8\pi^2\sqrt{2}}\left\{\frac{N_c}{3}K_f m_t^2 + M_W^2\left(\frac{3}{s_w^2}\ln c_w^2 - 5\right) + M_Z^2\left[\frac{1}{2} - 3\left(1 - 4s_w^2|Q_f|\right)^2\right]\right\}, \quad (1)$$

where $c_w^2 = 1 - s_w^2 = M_W^2/M_Z^2$, $K_b = 1$, and $K_f = 7$ for all other flavours, except for top. The $t\bar{t}$ mode will not be probed experimentally anytime soon and we shall not be concerned with it in the remainder of this paper.

Throughout this paper, we adopt the so-called modified on-mass-shell (MOMS) scheme [19], which emerges from the ordinary electroweak on-mass-shell scheme [17,20] by eliminating α in favour of G_F by virtue of the relation $G_F = \left(\pi\alpha/\sqrt{2}s^2 M_W^2\right)(1 - \Delta r)^{-1}$ [21]. Here, Δr embodies the non-photonic correction to the muon decay width. In the Born approximation of the MOMS scheme, one has [22]

$$\Gamma_0\left(H \to f\bar{f}\right) = \frac{N_c G_F M_H m_f^2}{4\pi\sqrt{2}}\left(1 - \frac{4m_f^2}{M_H^2}\right)^{3/2}, \quad (2)$$

where $N_c = 1$ for lepton flavours, and the weak correction is implemented by including the overall factor $(1 + \Delta_{\text{weak}})$.

Equation (1) has been obtained by putting $M_H = m_f = 0$ $(f \neq t)$ in the expression for the full one-loop weak correction. It provides a very good approximation for $f = \tau$ up to $M_H \approx 135$ GeV and for $f = b$ up to $M_H \approx 70$ GeV, the relative deviation from the full weak correction being less than 15% in each case. From Eq. (1) it is evident that the dominant effect is due to virtual top quarks. In the case $f \neq b$, the m_t dependence is carried solely by the renormalizations of the wave function and the vacuum expectation value of the Higgs field and is thus flavour independent. These corrections are of the same nature as those considered in Ref. [16]. For $f = b$, there are additional m_t dependent contributions from the $b\bar{b}H$ vertex correction and the b-quark wave-function renormalization. Incidentally, they cancel almost completely the universal m_t dependence. It is amusing to observe that a similar situation has been encountered in the context of the $Z \to f\bar{f}$ decays [23]. In summary, the universal virtual-top-quark term will constitute the most important part of the weak one-loop corrections to Higgs-boson decays in the near future. In this paper, we shall present the two-loop gluon corrections to this term.

This paper is organized as follows: In Sect. 2, we shall consider the universal contribution to the $H \to f\bar{f}$ decay rates induced by a pair of quarks with arbitrary masses and evaluate its QCD correction adopting the on-shell definition of quark mass. In Sect. 3, we shall repeat this calculation in the modified minimal-subtraction ($\overline{\text{MS}}$) scheme [24] and compare the result with the one of Sect. 2 in order to estimate the theoretical uncertainty related to the arbitrariness of the definition of quark mass. Section 4 contains the numerical analysis. Our conclusions are summarized in Sect. 5.

2   Two-loop results

In this section, we shall present the QCD correction to the shift in $\Gamma\left(H \to f\bar{f}\right)$ induced by a pair of quarks, (U, D), with arbitrary masses. For simplicity, we shall assume that U and D do not mix with f. Here, we shall adopt the on-shell definition of quark mass. As explained in Sect. 1, in the MOMS scheme, such corrections reside inside the renormalizations of the wave function and the vacuum expectation value of the Higgs field. The relevant part of Δ_{weak} is

$$\delta = -\frac{\Pi_{WW}(0)}{M_W^2} - \Re e \Pi'_{HH}\left(M_H^2\right), \tag{3}$$

where $\Pi_{WW}(s)$ and $\Pi_{HH}(s)$ are the unrenormalized self-energies of the W and Higgs bosons, respectively, evaluated at four-momentum squared s. In the following, we shall write down only the (U, D) contributions to the quantities under consideration.

For completeness, we shall first review the one-loop results. In dimensional regularization, they read [25]

$$\begin{aligned}
\Pi^0_{HH}(s) &= N_c \frac{G_F}{2\pi^2\sqrt{2}} \sum_{Q=U,D} m_Q^4 \left[\left(\frac{s}{2m_Q^2} - 3\right)\left(\frac{1}{\epsilon} - \ell_Q\right) - \left(\frac{s}{m_Q^2} - 4\right) f\left(\frac{s}{4m_Q^2}\right)\right. \\
&\quad \left. + \frac{s}{m_Q^2} - 5\right],
\end{aligned} \tag{4}$$

$$\Pi_{WW}^0(0) \;=\; N_c \frac{G_F M_W^2}{2\pi^2\sqrt{2}} \left[-\frac{1}{2} \sum_{Q=U,D} m_Q^2 \left(\frac{1}{\epsilon} - \ell_Q + \frac{1}{2} \right) + \frac{m_U^2 m_D^2}{2\left(m_U^2 - m_D^2\right)} \ln \frac{m_U^2}{m_D^2} \right], \qquad (5)$$

where

$$f(r) = \begin{cases} \sqrt{1 - \dfrac{1}{r}} \; \mathrm{arsinh}\sqrt{-r} & : \quad r < 0 \\[2ex] \sqrt{\dfrac{1}{r} - 1} \; \arcsin\sqrt{r} & : \quad 0 < r < 1 \;, \\[2ex] \sqrt{1 - \dfrac{1}{r}} \left(\mathrm{arcosh}\sqrt{r} - i\dfrac{\pi}{2} \right) & : \quad r > 1 \end{cases} \qquad (6)$$

$n = 4 - 2\epsilon$ is the dimensionality of space-time, and $\ell_Q = \ln\left(m_Q^2/\mu^2\right)$, with μ being the 't Hooft mass. Here and in the following, we suppress terms containing $\gamma_E - \ln(4\pi)$, where γ_E is Euler's constant. These terms may be retrieved by substituting $\mu^2 \to 4\pi e^{-\gamma_E}\mu^2$. When the on-shell definition of mass is employed, μ will drop out in the expressions for physical quantities and so will these terms. In the $\overline{\mathrm{MS}}$ scheme, these terms are subtracted along with the poles in ϵ. Inserting Eqs. (4,5) in Eq. (3), one obtains [18]

$$\begin{aligned} \delta_0 \;=\; N_c \frac{G_F}{2\pi^2\sqrt{2}} &\left\{ \sum_{Q=U,D} m_Q^2 \left[\left(1 + \frac{2m_Q^2}{M_H^2} \right) \Re e f\left(\frac{M_H^2}{4m_Q^2} \right) - \frac{1}{4} - \frac{2m_Q^2}{M_H^2} \right] \right. \\ &\left. - \frac{m_U^2 m_D^2}{2\left(m_U^2 - m_D^2\right)} \ln \frac{m_U^2}{m_D^2} \right\}. \end{aligned} \qquad (7)$$

In the limit $m_U, m_D \gg M_H/2$, this simplifies to [16]

$$\delta_0 = N_c \frac{G_F}{2\pi^2\sqrt{2}} \left[\sum_{Q=U,D} m_Q^2 \left(\frac{7}{12} - \frac{M_H^2}{10 m_Q^2} + \mathcal{O}\left(\frac{M_H^4}{m_Q^4} \right) \right) - \frac{m_U^2 m_D^2}{2\left(m_U^2 - m_D^2\right)} \ln \frac{m_U^2}{m_D^2} \right]. \qquad (8)$$

Setting $m_U = m_t$ and $m_D = 0$, one recovers the m_t-dependent term of Eq. (1). Light quarks, with $m_U, m_D \ll M_H/2$, decouple from δ.

We now proceed to two loops. The QCD correction to the Higgs-boson self-energy arises from the Feynman diagrams depicted in Fig. 1. Apart from the two-loop gluon-exchange diagrams [see Fig. 1(a)], one needs to include the diagrams where the one-loop quark mass counterterm, δm_Q, is inserted into the vertices and propagators of the one-loop seed diagram [see Fig. 1(b)].

In the on-shell scheme of mass renormalization, δm_Q is adjusted in such a way that the pole of the renormalized propagator appears at the renormalized mass. Specifically,

$$\frac{\delta m_Q}{m_Q} = \frac{1}{4m_Q^2} \mathrm{tr}(\not{p} + m_Q)\Sigma(p) \bigg|_{p^2 = m_Q^2}, \qquad (9)$$

where

$$\Sigma(p) = i4\pi\alpha_s C_F \left(\frac{\mu^2 e^{\gamma_E}}{4\pi} \right)^\epsilon \int \frac{d^n q}{(2\pi)^n} \frac{1}{q^2 + i\varepsilon} \gamma^\mu \frac{1}{\not{q} + \not{p} - m_Q + i\varepsilon} \gamma_\mu \qquad (10)$$

4

is the quark self-energy due to the exchange of one virtual gluon. As before, the combination $\gamma_E - \ln(4\pi)$ has been absorbed into a redefinition of the 't Hooft mass. A straightforward calculation yields [26]

$$
\begin{aligned}
\frac{\delta m_Q}{m_Q} &= -\frac{\alpha_s}{4\pi} C_F \left(\frac{\mu^2 e^{\gamma_E}}{m_Q^2} \right)^\epsilon \frac{3 - 2\epsilon}{\epsilon(1 - 2\epsilon)} \Gamma(1 + \epsilon) \\
&= -\frac{\alpha_s}{4\pi} C_F \left[\frac{3}{\epsilon} - 3\ell_Q + 4 + \epsilon \left(\frac{3}{2}\ell_Q^2 - 4\ell_Q + \frac{3}{2}\zeta(2) + 8 \right) + \mathcal{O}(\epsilon^2) \right].
\end{aligned} \tag{11}
$$

Using a notation consistent with Ref. [26], we find

$$
\Pi_{HH}^1(s) = N_c C_F \frac{G_F}{2\pi^2 \sqrt{2}} \frac{\alpha_s}{\pi} \sum_{Q=U,D} m_Q^4 \left[\frac{9}{2}X_1 - \left(\frac{s}{4m_Q^2} - 3 \right) Y_1 - \frac{s}{4m_Q^2} + 9\zeta(3) + H_1 \left(\frac{s}{4m_Q^2} \right) \right], \tag{12}
$$

where X_1 and Y_1 are the divergent constants that have been introduced in connection with the gauge-boson vacuum polarizations at $\mathcal{O}(\alpha\alpha_s)$ [27,28] and H_1 is a finite function. The expressions for X_1 and Y_1 depend on the regularization scheme. In dimensional regularization, they read[1]

$$
\begin{aligned}
X_1 &= \frac{1}{2\epsilon} - \ell_Q - 4\zeta(3) + \frac{55}{12}, \\
Y_1 &= \frac{3}{2\epsilon^2} + \frac{1}{\epsilon} \left(-3\ell_Q + \frac{11}{4} \right) + 3\ell_Q^2 - \frac{11}{2}\ell_Q + 6\zeta(3) + \frac{9}{2}\zeta(2) - \frac{11}{8}.
\end{aligned} \tag{13}
$$

For $r < 0$,

$$
\begin{aligned}
H_1(r) &= (r - 1)\left(2 - \frac{1}{r} \right) \left[6\,\mathrm{Li}_3\left(r_-^2 \right) - 3\,\mathrm{Li}_3\left(r_-^4 \right) + 8f\left(\mathrm{Li}_2\left(r_-^2 \right) - \mathrm{Li}_2\left(r_-^4 \right) \right) + 4f^2(-3f \\
&\quad + g + 2h) \right] + 4(r - 1)\sqrt{1 - \frac{1}{r}} \left[\mathrm{Li}_2\left(r_-^2 \right) - \mathrm{Li}_2\left(r_-^4 \right) + f(-3f + 2g + 4h) \right] \\
&\quad + f^2\left(-6r + 2 + \frac{13}{4r} \right) + 3f\left(-3r + \frac{7}{2} \right)\sqrt{1 - \frac{1}{r}} - \frac{3\zeta(3)}{r} + 3\zeta(2)(r - 3) + \frac{7}{4}, \tag{14}
\end{aligned}
$$

where Li_2 and Li_3 are the dilogarithm and trilogarithm [30], respectively, $r_\pm = \sqrt{1 - r} \pm \sqrt{-r}$, $f = \ln r_+$, $g = \ln(r_+ - r_-)$, and $h = \ln(r_+ + r_-)$. A table of handy transformation rules for the analytic continuation in r is avaliable from Ref. [31]. For $r > 1$, H_1 develops an imaginary part, which, by Cutkosky's rule [32], is related to the cuts of the two-loop amplitudes. This provides the opportunity for a nontrivial check of the calculation, since this imaginary part is related to the well-known $\mathcal{O}(\alpha_s)$ correction to $\Gamma(H \to q\bar{q})$. In fact, for $M_H > 2m_q$, one verifies that

$$
\Im m H_1 \left(\frac{M_H^2}{4m_q^2} \right) = \frac{\pi}{2} \frac{M_H^2}{m_q^2} \left(1 - \frac{4m_q^2}{M_H^2} \right)^{3/2} \delta_{\mathrm{QED}}, \tag{15}
$$

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[1] As a consequence of a misprint in Ref. [29], the term of Y_1 involving $\zeta(2)$ occurs in Ref. [28] with a wrong prefactor. Fortunately, this is inconsequential for the physical results of Ref. [28] because the Y_1 terms cancel exactly among themselves.

where δ_{QED} is given by Eq. (3.9) of Ref. [18].

The gluon correction to the (U, D) contribution to $\Pi_{WW}(0)$ may be found in Ref. [33], where the on-shell definition of quark mass is employed. The result may be written as

$$\Pi^1_{WW}(0) = \frac{N_c C_F}{4} \frac{G_F M^2_W}{2\pi^2 \sqrt{2}} \frac{\alpha_s}{\pi} \left[\sum_{Q=U,D} m^2_Q \left(Y_1 - 6\zeta(3) - 3\zeta(2) + \frac{23}{4} \right) + F\left(m^2_U, m^2_D\right) \right], \quad (16)$$

where

$$F(u, d) = (u - d)\,\text{Li}_2\left(1 - \frac{d}{u}\right) + \frac{d}{u - d} \ln\frac{u}{d} \left[u - \frac{3u^2 + d^2}{2(u - d)} \ln\frac{u}{d} \right]. \quad (17)$$

Note that $F(u, d) = F(d, u)$. From Eq. (17), we may read off the properties $F(u, u) = -u$ and $F(u, 0) = \zeta(2)u$. For $m_D = 0$, Eq. (16) coincides with Eq. (12) of Ref. [26].

Inserting Eqs. (12,16) into Eq. (3), we obtain the general expression for the (U, D) contribution to $\Gamma\left(H \to f\bar{f}\right)$ at next-to-leading order,

$$\delta_1 = \frac{N_c C_F}{4} \frac{G_F}{2\pi^2 \sqrt{2}} \frac{\alpha_s}{\pi} \left[\sum_{Q=U,D} m^2_Q \left(6\zeta(3) + 3\zeta(2) - \frac{19}{4} - \Re e H'_1\left(\frac{M^2_H}{4m^2_Q}\right) \right) - F\left(m^2_U, m^2_D\right) \right]. \quad (18)$$

For the reader's convenience, we list $\Re e H'_1$ for positive argument. For $0 < r < 1$, one has

$$\begin{aligned} H'_1(r) &= \left(2 - \frac{1}{r^2}\right)\left[6\,\text{Cl}_3(2\Phi) - 3\,\text{Cl}_3(4\Phi) + 8\Phi\,(\text{Cl}_2(2\Phi) - \text{Cl}_2(4\Phi)) - 4\Phi^2(\gamma + 2h)\right] \\ &\quad + \frac{4}{r}\sqrt{\frac{1}{r} - 1}\left[-\text{Cl}_2(2\Phi) + \text{Cl}_2(4\Phi) + 2\Phi(\gamma + 2h)\right] + \Phi^2\left(-6 + \frac{10}{r} + \frac{5}{4r^2}\right) \\ &\quad - \Phi\left(3 + \frac{25}{2r}\right)\sqrt{\frac{1}{r} - 1} + \frac{3\zeta(3)}{r^2} + 3\zeta(2) - \frac{9}{2} + \frac{21}{4r} \end{aligned} \quad (19)$$

and, for $r > 1$,

$$\begin{aligned} \Re e H'_1(r) &= \left(2 - \frac{1}{r^2}\right)\left[6\,\text{Li}_3\left(-\rho^2_-\right) - 3\,\text{Li}_3\left(\rho^4_-\right) + 8\phi\left(\text{Li}_2\left(-\rho^2_-\right) - \text{Li}_2\left(\rho^4_-\right)\right)\right. \\ &\quad + 2(2\phi^2 - 3\zeta(2))(-3\phi + \gamma + 2\chi)\Big] + \frac{2}{r}\sqrt{1 - \frac{1}{r}}\left[2\,\text{Li}_2\left(-\rho^2_-\right) - 2\,\text{Li}_2\left(\rho^4_-\right)\right. \\ &\quad - 9\zeta(2) + 2\phi(-3\phi + 2\gamma + 4\chi)\Big] + \phi^2\left(6 - \frac{10}{r} - \frac{5}{4r^2}\right) \\ &\quad - \phi\left(3 + \frac{25}{2r}\right)\sqrt{1 - \frac{1}{r}} + \frac{3\zeta(3)}{r^2} + 3\zeta(2)\left(-2 + \frac{5}{r} + \frac{5}{8r^2}\right) - \frac{9}{2} + \frac{21}{4r}, \quad (20) \end{aligned}$$

where Cl_2 and Cl_3 are the (generalized) Clausen functions of second and third order [30], respectively, $\Phi = \arcsin\sqrt{r}$, $\rho_\pm = \sqrt{r} \pm \sqrt{r - 1}$, $\phi = \ln\rho_+$, $\gamma = \ln(\rho_+ + \rho_-)$, and $\chi = \ln(\rho_+ - \rho_-)$. It is useful to know the expansions of H'_1 appropriate to the various limiting cases. They are

$$H'_1(r) = 6\zeta(3) + 3\zeta(2) - \frac{13}{4} + \frac{122}{135}r + \mathcal{O}(r^2) \quad (21)$$

in the heavy-quark limit $(r \ll 1)$,

$$H_1'(r) = \zeta(2)\left(-12h - 6\ln 2 + \frac{87}{8}\right) - \frac{9}{2}\zeta(3) + \frac{3}{4} + 3\pi\sqrt{1-r} + \mathcal{O}\big(h(1-r)\big) \qquad (22)$$

at threshold $(r \lesssim 1)$, and

$$\Re e H_1'(r) = 6\gamma^2 - 3\gamma - 6\zeta(2) - \frac{9}{2} + \frac{3}{r}(-6\gamma + 1) + \mathcal{O}\left(\frac{\gamma^2}{r^2}\right) \qquad (23)$$

in the light-quark limit $(r \gg 1)$. From the above results we can glean the leading QCD correction to K_f for $f \neq b$. Inserting Eq. (21) into Eq. (18), with $m_U = m_t \gg M_H/2$ and $m_D = 0$, and comparing the result with Eq. (1), one finds the corrected value,

$$\begin{aligned}
K_f &= 7 - 3\left(\zeta(2) + \frac{3}{2}\right)C_F\frac{\alpha_s}{\pi} \\
&= 7 - 2\left(\frac{\pi}{3} + \frac{3}{\pi}\right)\alpha_s \\
&\approx 7 - 4.004\,\alpha_s,
\end{aligned} \qquad (24)$$

where terms of $\mathcal{O}\left(M_H^2/m_t^2\right)$ have been suppressed. We recover the notion that, in Electroweak Physics, the one-loop $\mathcal{O}\left(G_F m_t^2\right)$ terms get screened by their QCD corrections.

3   Dependence on the quark-mass definition

So far, we have employed the on-shell definition of quark mass, i.e., we have evaluated the counterterm diagrams of Fig. 1(b) and similar diagrams for the W-boson self-energy using δm_Q in the form specified in Eq. (11). This is certainly a reasonable choice. In the approximation of neglecting the p^2 dependence of the imaginary part of the quark self-energy, the on-shell mass coincides with the real part of the complex pole position, i.e., with the physical mass, which is a constant of nature [34]. In general, the physical mass is close to what is determined experimentally, e.g., in quarkonium spectroscopy. In the case of the top quark, it is approximately the physical mass that is being extracted at the Tevatron and will be at future e^+e^- linear colliders, since, in the propagation of the t and $\bar{t}$ quarks between the production and decay vertices, configurations near the mass shell are greatly enhanced kinematically. As a matter of principle, however, this mass convention is arbitrary, and one might as well adopt another one. For, when the perturbation series is summed, the final result should not depend on the selected scheme. Yet, this holds no longer true when the perturbation series is truncated. In general, the finite-order results depend also on the renormalization scales of the quark masses. Scheme and typical scale variations may be used to estimate the theoretical uncertainty due to the unknown higher-order corrections.

In perturbative-QCD calculations, the quark masses are frequently defined according to the $\overline{\text{MS}}$ scheme. In this case, δm_Q collects just the pole in ϵ,

$$\frac{\delta\overline{m}_Q}{m_Q} = -\frac{\alpha_s}{4\pi}C_F\frac{3}{\epsilon}. \qquad (25)$$

The relationship between the on-shell mass and the $\overline{\text{MS}}$ mass, $\overline{m}_Q$, may be read off from Eq. (11) [35],

$$\overline{m}_Q = m_Q \left[1 + \frac{\alpha_s}{4\pi} C_F (3\ell_Q - 4) + \mathcal{O}\left(\alpha_s^2\right) \right]. \tag{26}$$

The mass renormalization scale, μ, must be chosen judiciously according to the problem at hand so as to minimize higher-order corrections. The $\mathcal{O}\left(\alpha_s^2\right)$ term of Eq. (26) may be found in Ref. [36], but we shall not need it here.

In the following, we shall translate the results of Sect. 2 to the $\overline{\text{MS}}$ scheme. That is, we have to repeat the evaluation of the two-loop counterterm diagrams using $\delta\overline{m}_Q$ instead of δm_Q. To this end, we may exploit the fact that the corresponding expressions may be constructed from the one-loop amplitudes by variation, e.g.,

$$\delta\Pi_{HH}^1(s) = \sum_{Q=U,D} \delta m_Q \frac{\partial}{\partial m_Q} \Pi_{HH}^0(s), \tag{27}$$

and similarly for the $\overline{\text{MS}}$ scheme. Consequently, the $\overline{\text{MS}}$ version of Π_{HH}^1 may be obtained from Eq. (12) by including the term

$$\Delta\Pi_{HH}^1(s) = \sum_{Q=U,D} (\delta\overline{m}_Q - \delta m_Q) \frac{\partial}{\partial m_Q} \Pi_{HH}^0(s). \tag{28}$$

Since $\delta\overline{m}_Q - \delta m_Q$ is devoid of ultraviolet singularities, knowledge of the $\mathcal{O}(\epsilon)$ term of Π_{HH}^0 is not necessary. After carrying out this operation in Eqs. (4,5), we may represent the result by assigning shifts to the various items appearing in Eqs. (12,16),

$$\Delta X_1 = 0, \tag{29}$$

$$\Delta Y_1 = \frac{1}{\epsilon}(3\ell_Q - 4) - \frac{9}{2}\ell_Q^2 + 8\ell_Q - \frac{3}{2}\zeta(2) - 8, \tag{30}$$

$$\Delta H_1(r) = (3\ell_Q - 4)\left[f(2r - 5)\sqrt{1 - \frac{1}{r}} - 2r + \frac{9}{2} \right], \tag{31}$$

$$\Delta F\left(m_U^2, m_D^2\right) = (3\ell_U - 4)m_U^2 \left[\frac{m_D^2}{m_U^2 - m_D^2} \left(\frac{m_D^2}{m_U^2 - m_D^2} \ln \frac{m_U^2}{m_D^2} - 1 \right) - \frac{1}{2} \right] + (U \leftrightarrow D). \tag{32}$$

A complimentary set of shifts appropriate to the gauge-boson vacuum polarizations at arbitrary four-momentum may be found in Ref. [37]. Similarly to Eq. (4), Eq. (31) is valid for $r < 0$. To compute the shift of Eq. (18), we need $\Delta\Re e H_1'$ for $r > 0$. This is

$$\Delta H_1'(r) = (3\ell_Q - 4)\left[\frac{\Phi}{\sqrt{1/r - 1}}\left(-2 + \frac{1}{r} + \frac{5}{2r^2} \right) - 1 - \frac{5}{2r} \right] \tag{33}$$

for $0 < r < 1$ and

$$\Delta\Re e H_1'(r) = (3\ell_Q - 4)\left[\frac{\phi}{\sqrt{1 - 1/r}}\left(2 - \frac{1}{r} - \frac{5}{2r^2} \right) - 1 - \frac{5}{2r} \right] \tag{34}$$

for $r > 1$.

In the case of the (t, b) contribution to the fermionic decay widths of an intermediate-mass Higgs boson, we may set $m_U = m_t \gg M_H/2$ and $m_D = 0$. Then, the shift in Eq. (18) becomes

$$\Delta\delta_1 = \frac{N_c C_F}{4} \frac{G_F m_t^2}{2\pi^2\sqrt{2}} \frac{\alpha_s}{\pi} (3\ell_t - 4) \left(-\frac{7}{6} + \mathcal{O}\left(\frac{m_H^4}{m_t^4}\right)\right). \tag{35}$$

Thus, the $\overline{\text{MS}}$ version of Eq. (24) is given by

$$\overline{K}_f = 7 + \left(-3\zeta(2) + \frac{19}{2} - \frac{21}{2}\ell_t\right) C_F \frac{\alpha_s}{\pi}, \tag{36}$$

where terms of $\mathcal{O}\left(M_H^2/m_t^2\right)$ have been neglected. For $\mu = m_t$, this is $7 + (2/3)(19/\pi - \pi)\alpha_s \approx 7 + 1.938\,\alpha_s$. That is, the magnitude of the QCD correction is about half as large as in the on-shell case and its sign is opposite.

4 Numerical analysis

We are now in a position to explore the phenomenological consequences of our results. To start with, we specify the input values for our numerical analysis. We use $M_W = 80.24$ GeV [38], $M_Z = 91.1895$ GeV [39], $m_\tau = 1.777$ GeV [40], $m_b = 4.72$ GeV [41], and $m_t = (174 \pm 16)$ GeV [42]. We parameterize the hadronic contribution to the photon vacuum polarization according to Ref. [43], with the updated reference value $\Delta\alpha_{hadrons} = 0.0283$ at $\sqrt{s} = 91.175$ GeV [44], and use an equivalent set of effective mass parameters for the light quarks otherwise [43]. For $\alpha_s(\mu)$, we employ the $\overline{\text{MS}}$ formula of Ref. [45] and fix the asymptotic scale parameter appropriate to five active flavours, $\Lambda_{\overline{\text{MS}}}^{(5)}$, by requiring that $\alpha_s(M_Z) = 0.124$ [39]. Unless stated otherwise, we shall choose $\mu = m_t$ for (t, b) contributions and $\mu = M_H$ else. All other input parameters are adopted from Ref. [46].

In Fig. 2, we show versus M_H the radiative corrections to the leptonic decay widths of the Higgs boson originating from quark loops (dotted lines) and one-gluon exchanges within these loops for $m_t = (174 \pm 16)$ GeV. These corrections are mainly due to the (t, b) doublet and do not depend on the flavour of the final-state fermions. As we have observed already in Sect. 2, the QCD correction reduces the one-loop quark contribution. For $m_t = 176$ GeV, the screening effect ranges between 6.5% and 10.4% for M_H between 60 and 200 GeV.

The full electroweak correction does depend on the produced flavour, as may be seen already from the approximation of Eq. (1). Figure 2 presents the M_H dependence of the full electroweak one-loop [18] (dotted lines) plus QCD two-loop correction to $\Gamma(H \to \tau^+\tau^-)$ for $m_t = (174 \pm 16)$ GeV. Since the QED correction depends logarithmically on M_H, the slopes of the curves are steeper than in Fig. 2. At one loop, there is a large cancellation between the bosonic and fermionic contributions [18]. For $m_t = 174$ GeV, their sum is in fact negative and relatively small, so that the QCD correction enhances significantly the size of the total correction, by 50.0% (9.7%) at $M_H = 60$ GeV (150 GeV).

Figures 2,3 refer to the on-shell definition of quark mass. In Figs. 4,5, we study how the radiative corrections to the leptonic decay widths of the Higgs boson are affected when the quark mass renormalization is converted from the on-shell scheme to the $\overline{\text{MS}}$ scheme. For a meaningful study of the scale dependence, it is necessary to distinguish between the renormalization scales of the quark mass and the strong coupling constant, μ_m and μ_c,

respectively. The scale μ that occurs explicitly in the above formulae must be identified with μ_m. Since the quark contributions scale like m_q^2 [see Eqs. (7,18)], we may restrict our considerations to the top quark and neglect the masses of the other quarks. Figure 4 visualizes the one- and two-loop evaluations in the on-shell and $\overline{\text{MS}}$ schemes as a function of the top-quark on-shell mass, m_t, for $M_H = 100$ GeV. The solid and dashed lines represent the one- and two-loop on-shell results, respectively. The dot-dashed line corresponds to the one-loop evaluation with m_t replaced by $\overline{m}_t$ as given by Eq. (26) with $\mu_m = m_t$. Inclusion of $\delta_1 + \Delta\delta_1$, evaluated with $\overline{m}_t$, then leads to the dotted line. The difference between the dashed and dotted lines is hardly visible on the plot, which indicates that the scheme dependence at two loops and beyond is negligibly small.

In addition to the trivial μ_c dependence via α_s, which is present also in the on-shell scheme, the $\overline{\text{MS}}$ analysis depends on the mass renormalization scale, μ_m, too. A plausible choice is $\mu_m = m_t$, since this eliminates the terms proportional to ℓ_t. By the same token, it seems unnatural to choose μ_m very different from m_t, since this renders ℓ_t artificially large. In Fig. 5, we analyze the scale dependence of the two-loop $\overline{\text{MS}}$ result for $M_H = 100$ GeV and $m_t = 174$ GeV. As usual, we introduce a dimensionless scale parameter, ξ, which we vary from 1/4 to 4. The dotted line corresponds to the choice $\mu_c = \xi m_t$, $\mu_m = m_t$, which could be studied also in the on-shell scheme. Since the QCD corrections are known only to leading order, the ξ dependence is monotonic. It is expected to flatten when three-loop QCD corrections will be taken into account. The dashed line represents the evaluation with $\mu_c = m_t$, $\mu_m = \xi m_t$. Here, we observe a stabilization already at two loops. This is due to a cancellation between the μ_m dependence induced in the one-loop result via $\overline{m}_t$ and the one carried by the genuine two-loop term. There is no obvious reason to choose μ_m different from μ_c. When we identify the two scales and set $\mu_c = \mu_m = \xi m_t$, we obtain the solid curve, which assumes its maximum very close to one, at $\xi = 0.958$. This choice may thus be advocated by appealing to the principle of minimal sensitivity [47].

5 Conclusions

In the Standard Model, low- and intermediate-mass Higgs bosons decay preferably into fermion pairs, and the one-loop electroweak corrections to the respective decay rates are dominated by terms of $\mathcal{O}\left(G_F m_t^2\right)$. The leading two-loop corrections to these terms are of $\mathcal{O}\left(\alpha_s G_F m_t^2\right)$ and $\mathcal{O}\left(G_F^2 m_t^4\right)$. We have calculated the two-loop gluon correction to the shift in the fermionic decay rates of the Higgs boson induced by pairs of virtual quarks with arbitrary masses. This correction is of $\mathcal{O}\left(\alpha_s G_F m_Q^2\right)$, where m_Q is the mass of the heaviest quark, and does not depend on the produced fermion flavour. In the case of lepton-pair production, this is the only QCD correction up to two loops. In the case of the hadronic decays of the Higgs boson, this is the only source of $\mathcal{O}\left(\alpha_s G_F m_t^2\right)$ corrections. A special situation arises in the case of $b\bar{b}$ final states. Here, the universal $\mathcal{O}\left(G_F m_t^2\right)$ term of the electroweak one-loop correction is almost entirely cancelled by similar contributions to the $b\bar{b}H$ vertex and the b-quark wave function induced by the charged current. The two-loop $\mathcal{O}\left(\alpha_s G_F m_t^2\right)$ corrections to the latter contributions are currently under study [48].

For $M_H = 60$ GeV (200 GeV), QCD effects screen the positive universal $\mathcal{O}\left(G_F m_t^2\right)$ term by 6.5% (10.4%), provided the top quark mass is renormalized according to the on-shell

scheme. In the $\overline{\text{MS}}$ scheme, the QCD correction has a different sign and its magnitude is roughly half as large. However, the sum of the one- and two-loop corrections is very insensitive to the choice of mass renormalization scheme. In the $\overline{\text{MS}}$ evaluation with a common renormalization scale, μ, for the strong coupling constant and the top-quark mass, the total correction is stable under variations of μ in the vicinity of $\mu = m_t$. This indicates that the effect of QCD corrections beyond two loops is likely to be insignificant.

ACKNOWLEDGMENTS

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FIGURE CAPTIONS

Figure 1: Feynman diagrams pertinent to the QCD corrections to the Higgs-boson self-energy: (a) gluon exchanges and (b) counterterms.

Figure 2: $\mathcal{O}(\alpha)$ and $\mathcal{O}(\alpha\alpha_s)$ radiative corrections to $\Gamma\left(H \to \ell^+\ell^-\right)$, with $\ell = e, \mu, \tau$, due to virtual quarks as a function of M_H for $m_t = (174 \pm 16)$ GeV.

Figure 3: Full $\mathcal{O}(\alpha)$ and $\mathcal{O}(\alpha\alpha_s)$ radiative corrections to $\Gamma\left(H \to \tau^+\tau^-\right)$ as a function of M_H for $m_t = (174 \pm 16)$ GeV.

Figure 4: $\mathcal{O}(\alpha)$ and $\mathcal{O}(\alpha\alpha_s)$ radiative corrections to $\Gamma\left(H \to \ell^+\ell^-\right)$, with $\ell = e, \mu, \tau$, due to virtual quarks as a function of m_t for $M_H = 100$ GeV. The evaluations using the on-shell and $\overline{\text{MS}}$ definitions of the top-quark mass are compared. All other quark masses are set to zero.

Figure 5: $\mathcal{O}(\alpha\alpha_s)$ radiative corrections to $\Gamma\left(H \to \ell^+\ell^-\right)$, with $\ell = e, \mu, \tau$, for $M_H = 100$ GeV and $m_t = 174$ GeV evaluated in the $\overline{\text{MS}}$ scheme with $\mu_c = \xi m_t$, $\mu_m = m_t$ (dotted line), $\mu_c = m_t$, $\mu_m = \xi m_t$ (dashed line), and $\mu_c = \mu_m = \xi m_t$ (solid line). All other quark masses are set to zero.

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